5/2/16

"The grass is always greener on the other side until you have to mow it." -Unknown

HW: "Summation Notation" packet homework section #1-3

AIM: What is Summation Notation?

a= start
n= finish

**SUMMATION (SIGMA) NOTATION**

$$\sum_{i=a}^{n} f(i) = f(a)+f(a+1)+f(a+2)+\cdots+f(n)$$

where $i$ is called the **index variable**, which starts at a value of $a$, ends at a value of $n$, and moves by unit increments (increase by 1 each time).

*Exercise* **#1:** Evaluate each of the following sums.

end

(a) $\sum_{i=3}^{5} 2i$

Start with 3

$2(3)+2(4)+2(5)$

$6+8+10=\boxed{24}$

(b) $\sum_{k=-1}^{3} k^2$

$(-1)^2+(0)^2+(1)^2+(2)^2+(3)^2$

$1+0+1+4+9=15$

(c) $\sum_{j=-2}^{2} 2^j$

$2^{-2}+2^{-1}+2^0+2^1+2^2$

$\frac{1}{4}+\frac{1}{2}+1+2+4=\frac{31}{4}$

(d) $\sum_{i=1}^{5} (-1)^i$

$\sum_{\boxed{i}=\boxed{1}}^{\boxed{5}} ((-1)^i)$

$(-1)^1+(-1)^2+(-1)^3+(-1)^4+(-1)^5$

$-1+1+(-1)+1+(-1)=-1$

(e) $\sum_{k=0}^{2} (2k+1)$

$(2(0)+1)+(2(1)+1)+(2(2)+1)$

$1+3+5=9$

(f) $\sum_{i=1}^{3} i(i+1)$

$\sum_{\boxed{i}=\boxed{1}}^{\boxed{3}} (i(i+1))$

$(1(1+1))+(2(2+1))+(3(3+1))$

$2+6+12=\boxed{20}$

*Exercise* **#2:** Which of represents the value of $\sum_{i=1}^{4} \frac{1}{i}$?

(1) $\frac{1}{10}$ (3) $\frac{25}{12}$

(2) $\frac{9}{4}$ (4) $\frac{31}{24}$

$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$

$\frac{12}{12}+\frac{6}{12}+\frac{4}{12}+\frac{3}{12}=\frac{25}{12}$

***Exercise #3:*** Consider the sequence defined recursively by $a_n = a_{n-1} + 2a_{n-2}$ and $a_1 = 0$ and $a_2 = 1$. Find the value of $\sum_{i=4}^{7} a_i$

$a_4 + a_5 + a_6 + a_7$

$3 + 5 + 11 + 21$

$40$

$a_3 = 1 + 2(0) = 1$

$a_4 = 1 + 2(1) = 3$

$a_5 = 3 + 2(1) = 5$

$a_6 = 5 + 2(3) = 11$

$a_7 = 11 + 2(5) = 21$

***Exercise #4:*** Express each sum using sigma notation. Use $i$ as your index variable. First, consider any patterns you notice amongst the terms involved in the sum. Then, work to put these patterns into a formula and sum.

(a) $9 + 16 + 25 + \cdots + 100$

$3^2 + 4^2 + 5^2 \ldots\ldots 10^2$

$$\sum_{x=3}^{10} x^2$$

(b) $-6 + -3 + 0 + 3 + \cdots 15$

+3 +3 +3 ......

Add 3 every time (multiply)

$$\sum_{x=-2}^{5} 3x$$

$$\sum_{x=-3}^{4} 3x+3$$

(c) $\frac{1}{25} + \frac{1}{5} + 1 + 5 + \cdots 625$

$5^{-2} + 5^{-1} + 5^{0} + 5^{1} \ldots + 5^{4}$

$$\sum_{x=-2}^{4} 5^x$$

***Exercise #5:*** Which of the following represents the sum $3 + 6 + 12 + 24 + 48$?

×2 ×2 ×2 ×2

(1) $\sum_{i=1}^{5} 3^i$

(3) $\sum_{i=0}^{4} 6^{i-1}$

(2) $\sum_{i=0}^{4} 3(2)^i$

(4) $\sum_{i=3}^{48} i$

$3 \cdot 2^x$

***Exercise #6:*** Some sums are more interesting than others. Determine the value of $\sum_{i=1}^{99}\left(\frac{1}{i} - \frac{1}{i+1}\right)$. Show your reasoning. This is known as a **telescoping series (or sum)**.

$\left(\frac{1}{1} - \frac{1}{1+1}\right) + \left(\frac{1}{2} - \frac{1}{2+1}\right) + \left(\frac{1}{3} - \frac{1}{3+1}\right) + \left(\frac{1}{4} - \frac{1}{4+1}\right) \ldots$

$\left(1 - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}\right) \ldots \left(\cancel{\frac{1}{99}} - \frac{1}{100}\right)$

$\boxed{\left(1 - \frac{1}{100}\right)} = \frac{99}{100}$

HW finish the
packet